Some of these limitations have been taken care of in another accelerator known as *Van de Graaff accelerator*.

6.3 VAN DE GRAAFF ACCELERATOR

This accelerator was developed by Robert J. Van de Graaff in 1931 at Massachusetts Institute of Technology, USA and was initially capable to stand a potential difference of 5 million volts but later on it was upgraded to withstand a potential difference of 7 million volts.

6.3.1 Principle

Van de Graaff accelerator is based on the principle that if a charged conductor is brought in internal contact with another conductor, which is hollow, all of its charge gets transferred to the hollow conductor, no matter how high the potential is on the later. Therefore, by successively adding charge, the potential of the hollow conductor can be raised to any desired value.

6.3.2 Construction

It consists of a hollow spherical-shaped conductor, which is mounted on a long insulating support. The hollow conductor is also known as *high-voltage terminal* or *terminal* or *dome*. An insulating belt made up of silk, rubber or plastics, etc., passes over two frictionless pulleys, P_1 and P_2. Pulley P_1 is mounted on the grounded end of the structure, while second pulley P_2 is enclosed within spherical-shaped high-voltage terminal as shown in Figure 6.2. C_1 and C_2 are two sharp pointed combs. Comb C_1, called *spray comb*, charges the belt with charge sprayed by its sharp needles, close to the belt at the ground pulley. This comb is connected to a power supply that raises its potential to a few tens of kilovolts.

The comb C_2 is called charge collection comb and is placed near the belt inside the high-voltage terminal or dome and is connected to the inner side of the dome. An ion source is placed inside the dome and is connected to the accelerating tube A. On the other side of the accelerating tube, the target is placed in the vacuum. The entire accelerating tube is highly evacuated. This is to make sure that accelerated particles do not collide with air molecules and get scattered. Entire structure which includes high-voltage terminal, pulleys, belt, etc. is enclosed in a steel pressure tank (not shown in Figure 6.2) in

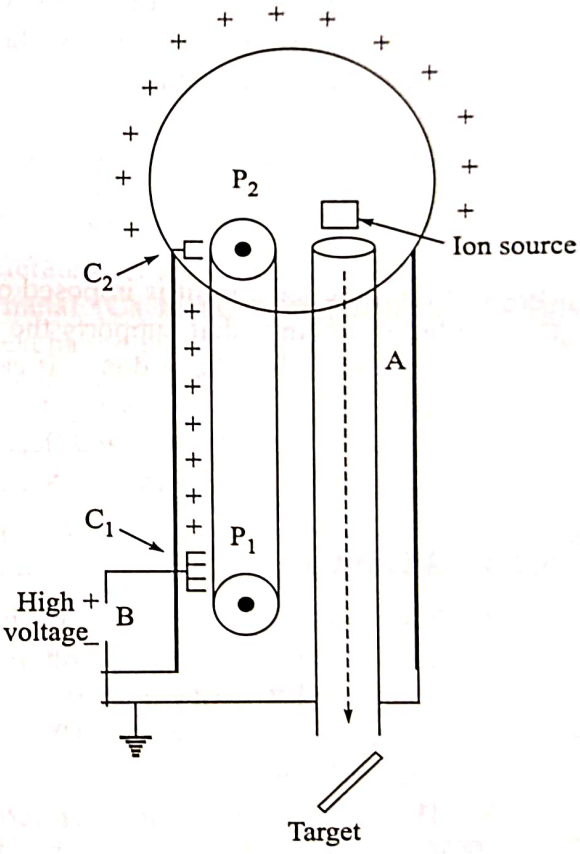

Figure 6.2 Schematic diagram of Van de Graaff accelerator.

which SF_6 gas is filled at pressure of the order of 10–20 atmospheres. SF_6 gas is chosen as it has much higher breakdown voltage compared to air. With SF_6 gas accelerators have been built where high-voltage terminal can withstand potential differences of the order of 20–30 MV.

6.3.3 Working

A dc potential of 5 kV to 50 kV is applied at a point B, the positive end is connected to the comb C_1 and negative end is grounded. The belt is run at high speed with the help of motor. The comb C_1 is connected to high-voltage supply, an intense electrostatic field is set up near the sharp points of the comb. This field produces positive and negative ions in the air. This discharge of air is also known as *Corona discharge*. (The electric field gradient at the surface of the conductor depends upon its radius of curvature. When a conductor having charge is a sharply pointed edge like comb, the radius of curvature of the pointed edge is very small. The charge density at the tip will be very large. So the pointed edge leaks the charge in the form of a discharge known as *corona discharge*. It sprays charge on the moving belt.) Sharp points of the comb C_1 attracts negative electrons and repel positive ions towards the fast-moving belt. The positive ions get deposited on the belt or belt becomes positively charged. As the positively charged belt moves up in the dome, it picks up electrons from the comb C_2 and the positive charge on the belt gets neutralized. As the comb C_2 is connected to the inner side of the dome, the inner side of the dome gets positive charge, which is immediately transferred to outer surface of the dome. The resulting potential V on the outer conductor can be calculated as

$$V = \frac{Q}{C}$$

where C is the total capacity and Q is the total charge on the conductor.

In principle, the potential can be increased without limits as we add more and more charge Q. However, in practice a limit is imposed on the potential by the electric breakdown (sparking) of the insulating column that supports the dome or of the surrounding atmosphere. When air is surrounding the high-voltage dome, it can stand voltages up to about 2–5 MV. However, if the entire accelerator is enclosed in SF_6 gas, it can stand voltages up to about 20 MV. When the applied potential difference is low, few hundreds of kV, as in small Van de Graaff accelerators, the gases used are dry N_2, CO_2, etc.

6.3.4 Advantages

1. The Van de Graaff accelerator has one enormous advantage over the Cockcroft Walton accelerator. The terminal voltage of Van de Graaff accelerator is highly stable. Terminal voltage is constant within ±0.1%. This means that the spread in the energy of accelerated particles is from 1 keV to 10 keV, when the energy of accelerated particles is between 1 and 10 MeV.
2. The energy of the accelerated particles is higher compared to Cockcroft Walton accelerator.

6.3.5 Limitations

1. One major limitation of Van de Graaff accelerator is its low-current output. The beam current in this accelerator is of the order of μA.
2. The ion source in this accelerator is also in the high-voltage area like that of Cockcroft Walton accelerator.

6.4 TANDEM ACCELERATOR

Modified form of Van de Graaff accelerator which eliminates the problem of placing ion source in high-voltage area, is known as *Tandem accelerator*. Tandem

The accelerators discussed above cannot be used to accelerate particles to very high energies because of the breakdown in the electrical insulation and sparking in the air. This difficulty was overcome by the discovery of an altogether different principle for accelerating particles. Using the principle of resonance, particles are accelerated in steps and in each step; particles acquire additional energy by application of a relatively small voltage. This technique has led to the development of two types of accelerators:

- Linear accelerator LINAC or (drift tube accelerator), and
- Cyclic accelerator or cyclotron.

6.5 LINEAR ACCELERATOR (LINAC) OR DRIFT TUBE ACCELERATOR

A linear particle accelerator (also called a LINAC) is an electrical device for the acceleration of subatomic particles. This type of particle accelerator has many applications, from the generation of X-rays in hospitals, to an injector into higher energy accelerators at a dedicated experimental particle physics laboratory. The design of a LINAC depends on the type of particle that is being accelerated: electron, proton or ion. They range in size from a cathode-ray tube to the 2-mile long Stanford Linear Accelerator Center in California.

LINACs of appropriate design are capable of accelerating heavy ions to energies exceeding those available in cyclic accelerators (to be discussed in Section 6.7), which are limited by the strength of the magnetic fields required to maintain the ions on a curved path. High power LINACs are also being developed for production of electrons at relativistic speeds, required since fast electrons travelling in an arc lose energy through synchrotron radiation; this limits the maximum power that can be imparted to electrons in a synchrotron of a given size.

6.5.1 Principle

Charged particles can be accelerated by allowing them to fall through a suitable potential difference.

6.5.2 Construction

Linear accelerator consists of a number of hollow metallic cylindrical electrodes of increasing length, arranged in a straight line as shown in Figure 6.4. Alternate electrodes, i.e., the first, third, fifth, etc. are joined to one terminal A and the second, fourth, sixth, etc. to other terminal B of a high frequency generator. In this arrangement alternate electrodes carry opposite electrical potentials. For example, in a particular half-cycle of the oscillation all the odd-numbered electrodes would be positive whereas those with even numbers would be negative. In the next half-cycle, the potentials are reversed.

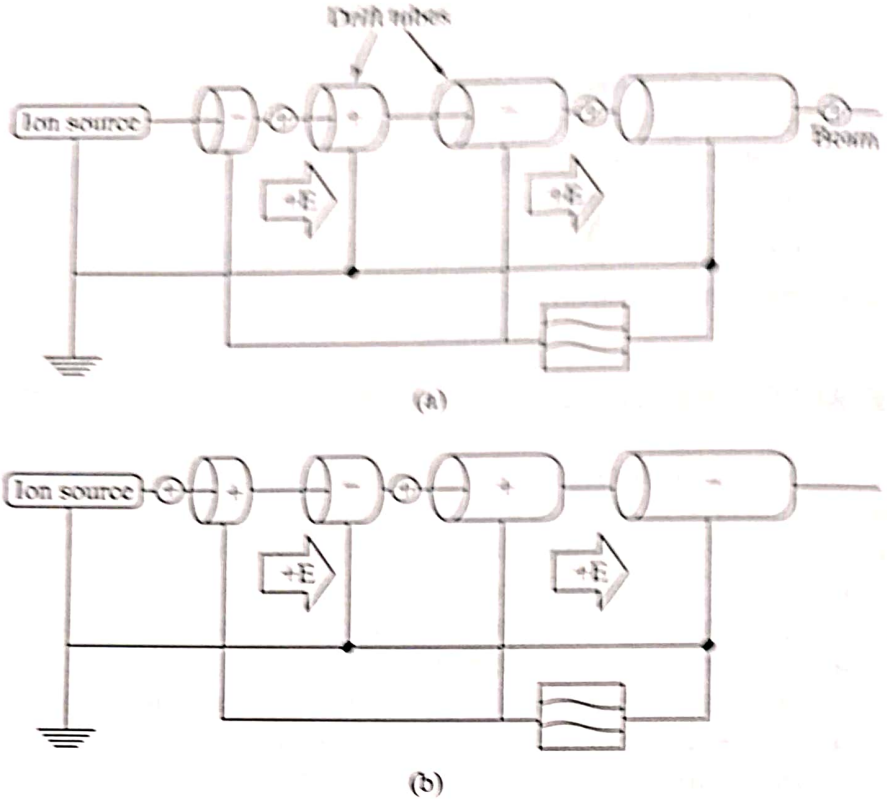

Figure 6.4 (a) the first drift tube is negatively charged. It means beam of +ve ions will be accelerated, (b) the first drift tube is positively charged. It means beam of +ve ions will not be accelerated.

6.5.3 Working

Suppose positively charged ions from the ion source S move from left to right along the common axis of the cylindrical electrodes. While passing through electrode 1, ions receive no acceleration, since they are moving in a field-free region. However, in traversing the gap between the 1st and 2nd electrodes, the ions are in the region in which there is a difference of potential. If the 1st electrode is positive and 2nd is negative, the positively charged ions are accelerated while crossing the gap. The positive ions then enter the 2nd electrode, and travel through it at a constant speed but at a higher velocity than in the 1st electrode. The length of this electrode is such that just as the ions reach the gap between 2nd and 3rd electrodes, the polarity of electrodes gets reversed, i.e. 2nd electrode becomes positive and 3rd becomes negative. This causes the positive ions to get additional acceleration in this gap also.

By making successive electrodes increasingly longer, to compensate for the increasing speed of the positive ions, it is possible to keep the ions in phase with the oscillating frequency. The positive ions gain energy each time they go from one electrode to the next.

If q is the charge on the ion, V_0 (volts) is the peak value of the ac frequency applied to the electrodes, then in each gap ions acquire energy = qV_0 eV. If there are n such gaps, then

$$\text{Energy acquired in } n \text{ gaps} = nqV_0 \text{ eV}. \tag{6.1}$$

The total kinetic energy acquired in n gaps $= \frac{1}{2}mv_n^2$.

Equating these two energies

$$\frac{1}{2}mv_n^2 = nqV_0$$

or

$$v_n = \sqrt{\frac{2nqV_0}{m}} \tag{6.2}$$

In these calculations, we have assumed that $v_n << c$.

Equation (6.2) shows that in order to get a high-energy beam

- the peak voltage V_0 of the oscillator should be higher.
- the number of gaps n should be as large as possible.

If f is the frequency of the oscillating potential, the time duration of a half-cycle is

$$= \frac{1}{2f}$$

The time ions take in passing through a nth cylinder of length L_n is

$$= \frac{L_n}{v_n}$$

where v_n is the velocity of the ions in nth cylinder.

For synchronization, we must have

$$\frac{L_n}{v_n} = \frac{1}{2f}$$

or

$$L_n = \frac{v_n}{2f}$$

Substituting for v_n from Eq. (6.2)

$$L_n = \frac{1}{2f}\sqrt{\frac{2nqV_0}{m}} \tag{6.3}$$

In 1931, Sloan and Lawrence built the first linear accelerator and using this accelerator they accelerated Hg ions to 2.85 MeV with peak oscillating voltage of 42 kV only.

Linear accelerators generally require very big set-ups. For example, LINAC at Los Almos in USA accelerating protons to 800 MeV is 805 metres long.

6.5.4 Advantages

1. Requirement of generating very high-voltages ~ million volts range is avoided in these accelerators.

2. They are economical for obtaining very high-energy particle beams.
3. They provide well-collimated beam of accelerated ions.

6.5.5 Limitations

1. They are inconveniently long in size.
2. They require extremely high frequency oscillators.

2. They are economical for obtaining very high-energy particle beams.
3. They provide well-collimated beam of accelerated ions.

6.5.5 Limitations

1. They are inconveniently long in size.
2. They require extremely high frequency oscillators.

6.6 WAVE-GUIDE ACCELERATORS

With drift tube accelerators discussed in Section 6.5, it is not possible to accelerate electrons. The reason for this is that in drift tube accelerators, lower frequencies ~ 300 MHz are used. At these frequencies, power utilization will be very poor and the length of the drift tubes would be impossibly long. Further, if very high frequencies > 300 MHz are used to provide power to the electron, the outer tube of the linear accelerator or cavity would be so narrow that it could not accommodate the drift tubes.

In order to accelerate electrons beyond ~ 2 MeV, alternate version of linear accelerator was based on wave-guides (a wave-guide is a hollow cylindrical metallic conducting tube). These accelerators are known as *wave-guide accelerators*. In such accelerators drift tubes are completely eliminated. These accelerators consist of a circular wave-guide loaded with metallic annular discs as shown in Figure 6.5. An electromagnetic wave travels along the axis of the tube. In order to reduce the phase velocity of the wave, which is normally higher than the velocity of light, to that of the electrons, discs are spaced at about one-third wavelength intervals. The electrons can be imagined as riding on the advancing wavefront of the electromagnetic wave as surfer riders ride on a water wave.

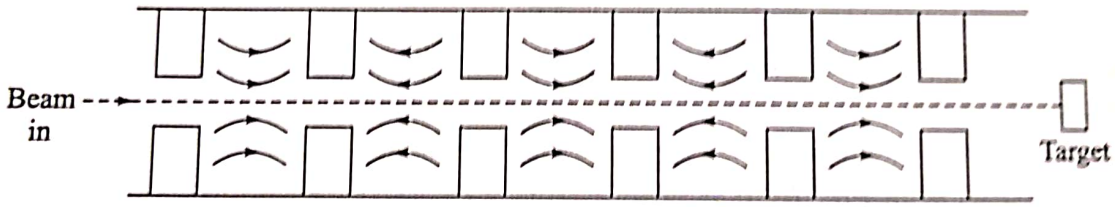

Figure 6.5 Resulting electric field in a disc loaded circular wave-guide accelerator.

Many electron linear accelerators based on this principle have been developed. The accelerator at Stanford University and that in the Russia produce 400 MeV electrons and the length of the accelerators are 79 and 110 metres respectively. A 20 GeV electron accelerator operational at Stanford University has a length of about 3 km.

6.7 MAGNETIC RESONANCE ACCELERATORS OR CYCLOTRONS

Because of the breakdown of electric insulation and sparking, particles cannot be accelerated to very high energies in electrostatic accelerators like Van de Graaff or Tandem accelerators.

Similarly, at very high energies, the length of linear accelerators becomes abnormally long (tens of kilometres). In 1930 Lawrence and Livingston suggested an alternative way of accelerating particles using a magnetic field to make the charged particles follow a spiral path of increasing radius. A beam of charged particles makes perhaps hundreds of revolutions, through the device and in each revolution particles receive two small energy increments until the particle energy reaches the desired value. This type of accelerators were initially called *magnetic resonance accelerators* but later named as *Cyclotrons*. In 1930, Lawrence and Livingston built the first experimental cyclotron, which was able to accelerate protons to 80 keV using an oscillator of peak voltage of only 2000 volts. The diameter of magnet pole pieces was 4 inch. Later on in 1932, another cyclotron was built which could accelerate protons to 1.2 MeV. It had a magnet with pole faces 11 inches in diameter. The diameter of the pole faces of the cyclotron is generally used to describe the size of the cyclotron. For example, if the diameter of poles of a magnet of a cyclotron is 88 inches, it would be referred as an "88-inch cyclotron". As we will see later in this section, larger the diameter of the pole face of the cyclotron magnet, greater would be the energy of the accelerated particles.

6.7.1 Principle

A positively charged particle can be accelerated to high energy with the help of an oscillating electric field, by making the particle to cross the same electric field time and again with the use of a strong magnetic field.

6.7.2 Construction

A cyclotron consists of two D-shaped hollow semicircular chambers D_1 and D_2 called "Dees" because of their shapes. These dees are placed with their diametric edges parallel to each other and are slightly separated from each other as shown in Figure 6.6. These dees are connected to the terminals of an alternating high frequency (10–20 MHz) and high voltage with peak value of 10–50 kV. This arrangement makes one dee positive and other negative during one half-cycle and vice versa in the next half-cycle. An ion source S of positive ions, such as protons, deuterons, α-particles, etc. is placed in the central region of the gap between the dees D_1 and D_2. This entire set-up, i.e. dees, ion source, etc. are placed in a highly evacuated chamber. The dees are completely insulated from this chamber. A uniform magnetic field is applied perpendicular to the cross-sectional area of the dees by placing them between the pole faces of a large electromagnet as shown in Figure 6.7.

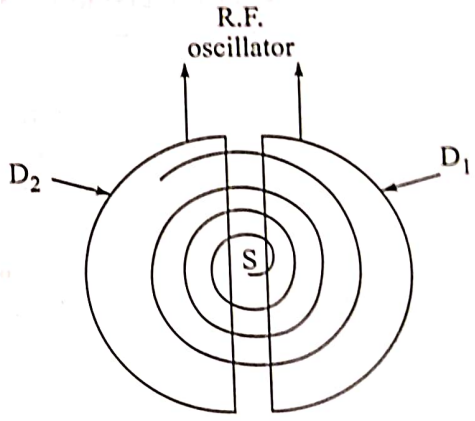

Figure 6.6 Cyclotron consisting of two dees placed in magnetic field.

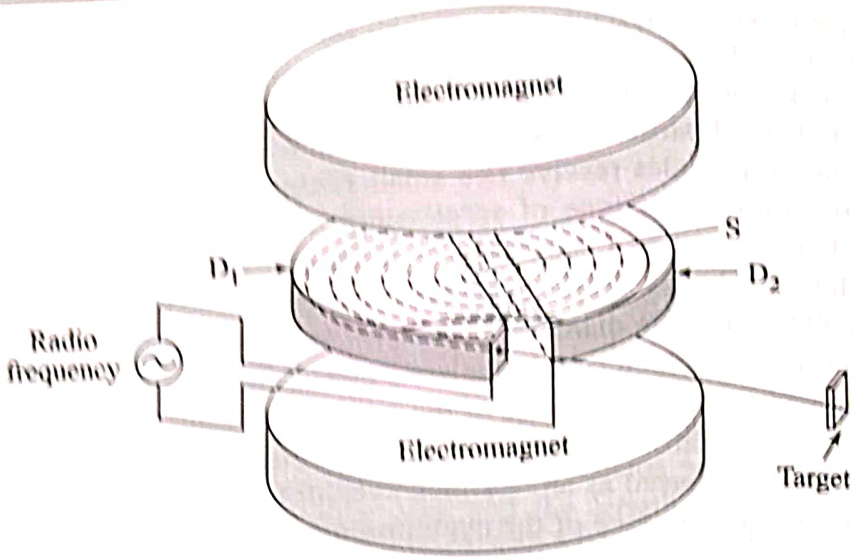

Figure 6.7 Detailed diagram of cyclotron.

6.7.3 Working

Ion source S produces positive ions within the gap between dees D_1 and D_2. At that particular instance, D_1 is positive and D_2 is negative. The positive ions are accelerated towards dee D_2 and it enters the dee D_2. As there is no electric field inside the dees, the ions are subjected to perpendicular magnetic field. The perpendicular magnetic field acts on the ions and the positive ions move in a circular path. Once inside the dee, they experience no acceleration and move with a constant speed. After traversing the semicircular path inside the dee D_2, they return to the gap between the dees. The frequency of the oscillator is adjusted in such a way that when ions reach the gap, the dee D_2 becomes positive and D_1 becomes negative. Now the positive ions are accelerated towards dee D_1, thus gaining in kinetic energy. The ions enter the dee D_1 with higher kinetic energy compared to the value when it was in dee D_2. In the dee D_1, they move again in a circular path with larger radius but with constant speed. After covering the semicircle inside the dee D_1, the ions reach the gap between the dees. At this instance again the polarity of the dees is reversed, positive ions again experience acceleration and this process is repeated many times (perhaps hundreds of times). Finally, when the positive ions reach the periphery of the dees after gaining a maximum energy, it is extracted out of the dees by means of high-voltage deflecting plates.

6.7.4 Theory

Let m be the mass of the positive ions to be accelerated, q is the charge on the ion. The ion moves in a semicircle of radius r with velocity v in a perpendicular magnetic field B.

Magnetic force acting on the ion $= qvB$

This force provides the necessary centripetal force $\frac{mv^2}{r}$ to the ions. Therefore,

$$\frac{mv^2}{r} = qvB$$

or

$$r = \frac{mv}{qB} \tag{6.4}$$

According to this equation, if m, q and B remain constant, then

$$r \propto v$$

As positive ions gain energy, i.e. v increases, the ions move in a semicircle of larger and larger radius. Therefore, the path of the ions is spiral in the dees.

If t is the time taken by the ions to complete a semicircular path, then

$$t = \frac{1}{2}\frac{2\pi}{\omega} = \frac{\pi}{\omega}$$

where ω is the angular frequency and $\omega = \frac{v}{r}$.

Therefore,

$$t = \frac{\pi r}{v}$$

Substituting r from Eq. (6.4)

$$t = \frac{\pi m}{Bq} \tag{6.5}$$

Time T to completing a full circular path is given by

$$T = 2t = \frac{2\pi m}{Bq}$$

Again if m, q and B are constant, then t, the time taken by the ion to complete the semicircular path inside the dee, is independent of the velocity of the ion, radius of the semicircular path and radius of dees or the size of pole pieces of the magnet. Hence, the frequency of the oscillations required to keep the ion in phase is given by

$$f = \frac{1}{T} = \frac{Bq}{2\pi m} \tag{6.6}$$

where f is known as cyclotron frequency or cyclotron resonance frequency and it is the frequency of ac oscillator also.

If $R = r_{max}$ is the radius of the last orbit from which the ions are extracted out of the dees and v_{max} is the velocity of the ions in the last orbit, then from Eq. (6.4)

$$R = \frac{mv_{max}}{qB}$$

or

$$v_{max} = \frac{BqR}{m} \tag{6.7}$$

Correspondingly, the maximum kinetic energy of the ions is

$$E_{max} = \frac{1}{2}mv_{max}^2$$

Substituting v_{max} from Eq. (6.7)

$$E_{max} = \frac{1}{2}\frac{mB^2q^2R^2}{m^2}$$

or

$$E_{max} = \frac{1}{2}\frac{B^2q^2R^2}{m} \tag{6.8}$$

The maximum kinetic energy acquired by the ions can also be calculated by multiplying the charge on the ion q, the peak value of the ac voltage V and the total number of circles N, completed by the ion before it is extracted out of the dees. Therefore,

$$E_{max} = NqV$$

Substituting E_{max} from Eq. (6.8)

$$\frac{1}{2}\frac{B^2q^2R^2}{m} = NqV$$

or

$$R = \frac{1}{B}\sqrt{\frac{2mV}{q}}N^{1/2} \tag{6.9}$$

From Eq. (6.8), it is clear that in order to accelerate particles to higher energy, we can

- increase B, i.e. make magnets with higher and higher pole strengths. But increasing B beyond a certain limit is not possible as most of the materials saturate at a particular magnetic field.
- increase R, i.e. build pole pieces of larger and larger diameters so that the particles are extracted at larger R.

6.7.5 Advantages

1. Cyclotron is much smaller in size compared to linear accelerators.
2. No high voltages (like in Van de Graaff accelerator) are required. Only low-voltage ac oscillator (10–50 kV peak value) is required.
3. Cyclotron can deliver tens of microamperes of current at the target.

6.7.6 Limitations

1. It has been estimated that the cost of building larger cyclotrons scales roughly as the cube of the energy. For example, the cost of 500 MeV cyclotron is about US$ 10^8. To build a cyclotron of 5 GeV is beyond the means of most of the countries.
2. As the energy of ions increases, relativistic effects come into picture. Because of the relativistic effects, mass of the ions m increases. m was assumed to be constant in Eqs. (6.4) and (6.5). Now it becomes a function of the velocity of the ions. This causes the ions to go out of phase with the applied ac oscillations and ions instead of accelerating may start de-accelerating. The upper limit of energy in case of protons is about 25–40 MeV.

3. Cyclotron cannot be used to accelerate electrons. The rest mass energy of electrons is very small (0.511 MeV); therefore, even for very low energies, mass of electrons does not remain constant, and they go out of phase with the applied ac voltage.

It was shown that allowance could be made for the effect of the increase of mass of the ions moving at high speed so as to keep them in phase with the oscillating potential. Two methods for this compensation are possible.

One method is to leave the magnetic field unchanged, but to decrease the frequency of the oscillating potential in accordance with Eq. (6.6). As the mass of the particle increases fm is kept constant. Cyclotrons based on this principle are known as *Synchrocyclotrons* or *Frequency Modulated Cyclotrons*.

The other method is to increase the magnetic field B in proportion to the mass, so that B/m remains constant. Due to this, the time period t in Eq. (6.5) remains unaffected, by the increase of mass. Accelerators based on this principle are known as *Sector Focusing or Azimuthally Varying Field* (*AVF*) Cyclotrons.

6.8 BETATRON

It is an accelerator for accelerating electrons or beta particles. There are definite problems associated with accelerating electrons by Van de Graaff and Cyclotron accelerators. The former machine could accelerate electrons up to a few MeV, while in case of latter machine relativistic effect becomes prominent. D.W. Kerst in 1940 built a new accelerator called *betatron*, which could accelerate electrons up to 250 MeV.

6.8.1 Principle

This machine uses the concept of electromagnetic induction as the accelerating force. It employs time varying magnetic field, which gives rise to induced electric field on the electrons moving in a fixed circular orbit and these electrons are accelerated. Here electrons move in a circular orbit of fixed radius. The rate of increase of magnetic flux (ϕ) is very slow compared to the frequency with which electrons are orbiting. In each orbit electrons are accelerated.

6.8.2 Construction

It consists of a doughnut-shaped chamber as shown in Figure 6.8, which is placed between the pole pieces of an electromagnet. The chamber is highly evacuated and the electrons with certain kinetic energy are injected into a circular orbit by an electron gun. By application of suitable electric and magnetic fields, electrons are forced to move in a constant radius orbit as shown by dotted line in Figure 6.8. When they attain full energy, they are deflected with the help of magnetic field on the target. This magnet and target are not shown in Figure 6.8 for making it simple. The electromagnet is powered by an alternating current. The inner layer of the chamber is coated with a thin layer of silver to avoid surface charge accumulation.

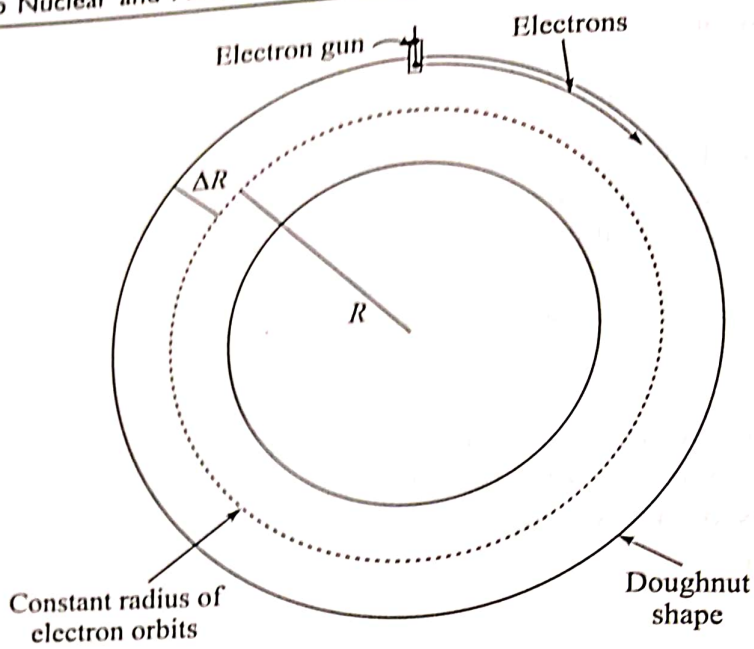

Figure 6.8 Schematic diagram of constant radius of electron orbits in a doughnut shaped betatron.

6.8.3 Working

The electrons are injected in the doughnut of the betatron during the first-quarter of a cycle in which magnetic field linking electron orbit is rising. Now to have electrons in a fixed orbit of radius r_0, a relation between the magnetic field at the orbit B and the total magnetic flux ϕ has to be derived.

An ac current having a frequency in the range 60–100 Hz powers the electromagnets. This generates a slow-varying field at the electron orbit of fixed radius.

The momentum of the electron $p = r_0 eB_0$

$$\text{The force acting on the electron} = \frac{d}{dt}(p) = \frac{d}{dt}(r_0 eB) = r_0 e\frac{dB}{dt} \tag{6.10}$$

Induced e.m.f. (ε) in the electron orbit is equal to work done by the unit charge in going around the orbit of radius r_0, i.e.

$$\varepsilon = \oint \vec{E} \cdot \vec{dl}$$

where E is the electric field which accelerates the electrons and from Faraday's laws ε is given by rate of change of magnetic flux.

$$\varepsilon = 2\pi r_0 E = \frac{d\phi}{dt} \tag{6.11}$$

The force on the electron is

$$eE = \frac{dp}{dt} \tag{6.12}$$

Combining Eqs. (6.10), (6.11) and (6.12)

$$\frac{d\phi}{dt} = 2\pi r_0^2 \frac{dB_0}{dt} \tag{6.13}$$

Integrating with respect to t, for ϕ varying from 0 to ϕ_0 and B varying from 0 to B_0 we get

$$\phi = 2\pi r_0^2 B_0 \tag{6.14}$$

This equation must be satisfied during the entire accelerating period, if the electrons have to be in the same orbit.

If B' is the average magnetic field over the whole area of the orbit, then

Total magnetic flux

$$\phi = \pi r_0^2 B' \tag{6.15}$$

Comparing Eqs. (6.14) and (6.15), we get

$$B_0 = \frac{1}{2} B' \tag{6.16}$$

This is known as *betatron condition*.

6.8.4 Average Energy per Orbit

The total flux ϕ varies as $\phi_0 \sin \omega t$ so the average kinetic energy gained per orbit E' in the first-quarter of the cycle, i.e.

$$\frac{T}{4} = \frac{\pi}{2\omega}$$

is given by the integral of the product of electron charge and induced e.m.f. over this time span. Mathematically, it is written as

$$E' = \frac{e\phi_0}{\pi/2\omega} \int_0^{\pi/2} \frac{d}{dt}(\sin \omega t) dt$$

or

$$E' = \frac{2e\omega\phi_0}{\pi} \tag{6.17}$$

Now

$$\phi_0 = 2\pi r_0^2 B_0$$

So, the average energy gained in one orbit is $E' = 4e\omega r_0^2 B_0$. (6.18)

6.8.5 Calculation of Final Energy of Electrons

Distance travelled by the electron in $T/4$ seconds = Velocity × time = $\frac{v\pi}{2\omega}$ (6.19)

where v is the velocity of the electron, which is fairly close to velocity of light.

Distance travelled by the electron in one orbit= $2\pi r_0$.

$$\text{Number of orbits that electron makes} = \frac{\text{Total distance travelled by the electron}}{\text{Distance travelled by the electron in one orbit}}$$

$$\text{Number of orbits } N_0 = \frac{v}{4\omega r_0} \tag{6.20}$$

Combining Eqs. (6.18) and (6.20),

$$\text{Total energy} = ver_0B_0 \tag{6.21}$$

As an illustration, we will discuss how betatron produces electrons with high energy.

Suppose electrons with energy 70 keV are introduced in the doughnut. The corresponding speed of the electrons comes out to be $\approx 2 \times 10^{10}$ cm/s. The radius of the orbit is 50 cm and if the electromagnet is powered by an ac frequency of 60 Hz and magnetic field at the orbit is 1 T, let us calculate:

1. Total distance travelled in $\frac{T}{4}$ seconds.
2. Number of orbits.
3. Average energy per orbit.
4. Total energy.

1. **Total distance travelled in $\frac{T}{4}$ seconds:** Using Eq. (6.19)

$$\text{Distance} = \frac{2\times10^{10}\pi}{2\times2\pi\times\nu}$$

$$= \frac{2\times10^{10}}{2\times2\times60}$$

$$= 8.33 \times 10^7 \text{ cm}$$

$$= 8.33 \times 10^5 \text{ m}$$

2. **Number of orbits:** Using Eq. (6.20)

$$\text{Number of orbits} = \frac{2\times10^8}{4\times2\times\pi\times\nu\times r_0}$$

$$= \frac{2\times10^8}{4\times2\times\pi\times60\times0.5}$$

$$= 2.65 \times 10^5 \text{ orbits}$$

3. **Average energy per orbit:** Using Eq. (6.18)

$$\text{Average energy per orbit} = 4 \times 1.6 \times 10^{-19} \times 2 \times \pi \times \nu \times 0.5^2 \times 1$$

$$= 4 \times 1.6 \times 10^{-19} \times 2 \times \pi \times 60 \times 0.5^2 \times 1$$

$$= 6.03 \times 10^{-17} \text{ J} \approx 375 \text{ eV}$$

4. **Total energy:** Using Eq. (6.21)

$$\text{Total energy} = ver_0B_0$$

$$= 2 \times 10^8 \times 1.6 \times 10^{-19} \times 0.5 \times 1$$
$$= 1.6 \times 10^{-11} \text{ J}$$
$$= 100 \text{ MeV}$$

6.9 SYNCHROCYCLOTRONS OR FREQUENCY MODULATED CYCLOTRONS

A synchrocyclotron is simply a cyclotron with the accelerating supply frequency decreasing as the particles become relativistic and begin to lag behind. Although in principle they can be scaled up to any energy they are not built any more as the synchrotron is a more versatile machine at high energies.

6.9.1 Principle

Synchrocyclotron is based on the principle that loss of resonance at high velocities, where there is an appreciable increase in the mass of the particle due to relativistic effects, can be compensated by decreasing the applied ac oscillating frequency.

6.9.2 Construction

The basic design of a synchrocyclotron is similar to that of a cyclotron. As shown in Figure 6.9, a synchrocyclotron consists of a single dee rather than two dees as in a conventional (low energy) cyclotron. Fixed magnetic field is applied with the help of pole pieces of an electromagnet, in a plane perpendicular to the plane of dee. The frequency of oscillating electric field applied is made to decrease continuously instead of keeping it constant so as to maintain the resonance with ion frequency. One terminal of the oscillating electric potential varying periodically is applied to the dee and the other terminal is earthed. The protons or deuterons to be accelerated are made to move in circles of increasing radii. The acceleration of particles takes place as they enter or leave the dee at the outer edge. The ion beam is taken out with the aid of electrostatic deflector.

The first synchrocyclotron built was at Berkeley, USA. It had magnets with pole piece diameter of 184 inches. In this synchrocyclotron, frequency of the oscillator was varied from 36 MHz to 18 MHz. Thirty-six megahertz was the initial frequency when the protons were moving with non-relativistic velocity and as they gained energy, the frequency was slowly reduced to 18 MHz. At this stage, protons would have gained maximum energy. This change in the frequency was done about 64 times per second. The magnetic field of the magnets is about 2.3 tesla. This accelerator was capable to accelerate protons to 740 MeV.

However, if we compare the output of cyclotron with that of synchrocyclotron, there is a difference. In a cyclotron, the flow of accelerated ions is regarded as continuous, although it actually consists of a series of pulses corresponding to each cycle of the oscillating potential. For a frequency of 20 MHz, there would be 20 million pulses of ions that reach the target per second. In the synchrocyclotron, however, a pulse of ions is carried from the ion source at the centre to the periphery of the dee as the frequency of the oscillating potential is decreased from its initial value (36 MHz) to the final value (18 MHz). The frequency then returns to its original high value (36 MHz) and another pulse of the ions is carried from the ion source to the periphery and so on.

The rate at which ion pulses are produced depends on the repetition rate of the frequency. This rate generally varies from 60 to 1000 per second, which means that at the target 60 to 1000 pulses of ions will be reaching the target per second. Because of this reason, the average value of the ion current is very low in these accelerators.

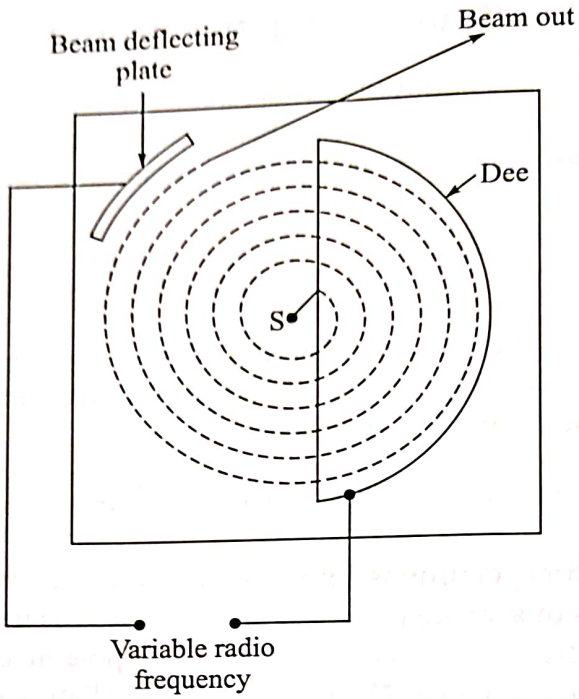

Figure 6.9 A schematic diagram of synchrocyclotron.

6.9.3 Theory

The frequency of revolution or angular frequency ω_c of the ions is

$$\omega_c = \frac{Bq}{m} = \frac{Bqc^2}{mc^2}$$

where mc^2 is the total energy of the ions which includes the kinetic energy T and the rest mass energy m_0c^2. Therefore,

$$\omega_c = \frac{Bqc^2}{T + m_0c^2}$$

Now $$\omega_c = 2\pi f \text{ or } f = \frac{\omega}{2\pi}$$

Here f is taken as the frequency of ac oscillator as it is in phase with the frequency of revolution of ions.

$$f = \frac{Bqc^2}{2\pi(T + m_0c^2)} \tag{6.22}$$

This frequency will have maximum value when $T \approx 0$, or

$$f_{max} = \frac{Bqc^2}{2\pi m_0 c^2} \tag{6.23}$$

and will have minimum value when $T \sim T_{max}$, or

$$f_{min} = \frac{Bqc^2}{2\pi (T_{max} + m_0 c^2)}$$

Solving these two equations for T_{max}, we get

$$T_{max} = \frac{f_{max} - f_{min}}{f_{max} f_{min}} \frac{Bqc^2}{2\pi} \tag{6.24}$$

6.9.4 Advantages

1. It is capable of accelerating positively charged particles to very high energies.
2. Normally, low power (~15 kW) oscillators are needed in these accelerators as a source of ac potential.
3. With one dee, the electrical and mechanical design become simple.

6.9.5 Limitation

1. The output beam current is very low around fraction of microamperes or even lower than that.

6.10 AZIMUTHALLY VARYING FIELD (A.V.F.) OR SECTOR FOCUSING CYCLOTRONS

It was shown that relativistic effects limited cyclotron energy because of incompatibility of resonance and focusing. Thomas pointed out that this conclusion applied only to azimuthally constant magnetic fields. If one allowed azimuthal variation the limit can be removed. This idea was not utilized by the accelerator builders until the late 1950s.

Thus, in early 1960s, a new type of cyclotron, the sector-focusing cyclotron emerged. Iron sectors were introduced in the pole gap so that an azimuthal variation of the magnetic field was obtained. This azimuthal variation provided a strong vertical focusing on the circulating beam of ions and it was then not necessary to have the azimuthally averaged field to decrease with increasing radius as it had to do in the conventional cyclotron in order to maintain vertical focusing. The frequency of the accelerating voltage can thus be kept constant while maintaining a steady acceleration at each gap traversal; the energy was limited only by the size of the magnet

6.10.1 Principle

The relativistic increase in mass of high velocity ions can be compensated by increasing th magnetic field B in proportion to the increase in mass m so that the ratio B/m remains constan Due to this the time period of the ion $t = \pi m/Be$ remains unaffected by increase in mass of the io